Analysis of algorithms' implementation for melodical operators in symbolical textual segmentation and connected evaluation of musical entropy

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Abstract: There are several algorithms that segment a musical piece, and process a list of melodic segments that are located thanks to some formal rules, which respect musical analysis. This article will present a melodic analysis pattern parsing in a progressive way the symbolic level of a musical text. The least informative unit is the note, to which a statistical approach is applied, when spreading the analysis to wider melodic segments, considering the single information carried by each one. The concept of information has been used for many years in the linguistic analysis and it has been applied to the musical language either.
This approach has been studied only focusing on the melodic field, avoiding concepts such as tonality, modulation and the rhythm particularly. The effectiveness of this model has been tested through the analysis of several pieces from different musicians and different ages, trying to vary different composition techniques: from monody to polyphony to harmonic writing, where the main melody is almost hidden inside polyphony, approaching different composition styles through a unique analysis methodology.

Key-Words: musical surface, melodic analysis, segmentation, entropy, melodic extraction, Musical Object

1 Analysis and information

Confronted by the amount of information daily received by anyone, analysis represents a central activity that models and interprets phenomena and reality processes.
Analysis becomes an instrument which allows a protection of a large flow of information and acts as a filter of this information.
Musically speaking, analysis provides reading ways that make understand the real essence of a musical event: listening to a work means both interpret and analyse it.
Analysis can change in every single listening and to every listener: it is the infinite plurality that is part of the opera, it is the expression of the work understanding in everyone's language, in their own emotions and expectations.
Facing this multiplicity, the aim is to work on the theories imposing models accepted by many listeners.
Even before it is crucial decide whether to presuppose the existence of universal rules that should be discovered from the job of analysis.
"Musical analysis is the subdivision of a musical structure into simpler elements (Musical Objects), and the research of the functions of these elements inside this structure".
By Musical Object it is meant is a staff fragment which contains a certain amount of notes, repeated several times (but also subject to variations, see Fig.2, 3, 4) in the same piece.

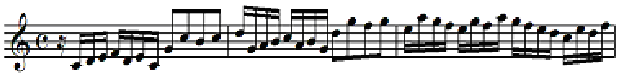

Fig.1: Invention with two voices from Bach n.1 in D Major (first three bars of the first staff).

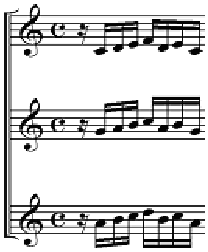

Fig.2: Invention with two voices from Bach n.1 in D Major. In the first stave it has been represented the notes sequence of the first bar , in the second and in the third one the transposition found regarding bar n 2 and n 8.

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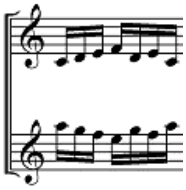

Fig.3: Inventions with two voices of Bach n.1 in D Major. In the first stave there is the notes sequence of bar 1, while in the second stave there is its inversion (bar 3).

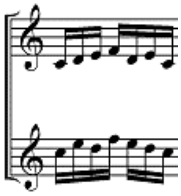

Fig.4: Invention with two voices of Bach n.1 in D Major. In the first stave we find the notes sequence of bar 1 instead in the second one there is its retrogradiation.

The main aim of this segmentation is to identify these musical objects that will be indicated as themes.
In musical analysis one of the most important elements is the object that represents the fundamental motif of a composition sometimes also recurring in a composition, especially if it is a long-time work.
It is a melodic fragment that has a clear individuality and recognition, often so important to characterize the whole musical piece.
Thematic elements are never such clearly expressed on the score by the musician, that is why a fragment does not have to be considered a motif.
This lack of explanation can identify the musical language as a meta-language, without rules about syntactic and semantic elements of the compositions, that make art mysterious.

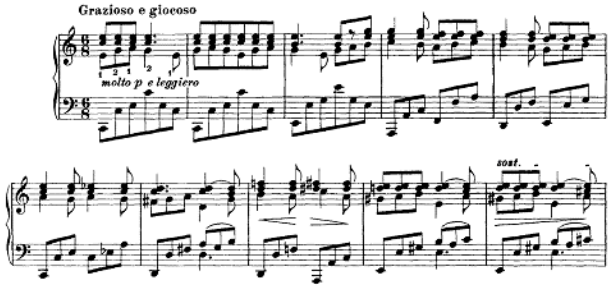

Fig.5: Extract from Intermezzo score Brahms op. 119 n. 3

The most important part of melodic analysis is to define what is intended by motif repetition; more specifically which of the textual repetitions in the score could be associated to the definition of musical object, when there are different repetitions, which criteria to adopt in order to select only the part that can be considered "meaningful" by an expert.

2 Information

Amongst the various algorithms realized for the melodic segmentation of a piece, the final choice of the segment is made through the comparison between each fragment found with a specifically compiled list of rules. The suggested model represents a new approach, based on a valuation of *information* that every musical fragment owns.
Information is the base of every human activity. All evolution, first biological, then social and cultural, has been marked by an ever increasing intense and diffuse exchange of information.
Information is the medium through which living beings evaluate the environment in which they live in to protect themselves, to feed themselves and to adapt.
This is why information is considered an important resource for life on the planet: without an exchanging of information, life could not have evolved or affirmed.
According to W. Weaver and C. Shannon, information concept includes:
symbolic apparatus through which it shows itself. Information is an abstract concept that realizes itself through a range of symbols, and it is the meaning assigned to those symbols that represent it, and meaning allows the interpretation of information.
Semantic content carried by information. The range of symbols carry a certain information data containing a precise meaning that signifies its value; this is the meaning that gives the semantic part of information. As far as it is concerned, it has to be underlined how information is sensitive to the state of the recipient: it loses value if it is already known to the receiver.
Pragmatic effect produced by information. Every information carries meanings with the intent of producing a certain effect.

"*Information is an ultimate un-definable or intuitive principle, whose precise definition seems in some way to slip through the fingers like a shadow*"

On the basis of the considerations made in the previous paragraph, it is possible to affirm that in a

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communication that happens through an alphabetical data of symbols, information is associated to every transmitted symbol.
This is why information can be defined as the reduction of uncertainty that anyone could have without the transmitted symbol.
In other words, the quantity of information contained in a message, is greater than the same degree as the level of unpredictability[1] in the content of the message itself.
The wider the message range is that the source can transmit (and greater the uncertainty to the receiver with respect to the possible message),the greater the quantity of information received, and along with it, its own measurement: the entropy.
In information theory, entropy is a positive value and not negative, as in physics.
Taken from order-from-disorder theories, entropy would be the engine of the species of living beings.
Mathematically, the measurement of the content of an information (I) is obtained from the Shannon formula :

$$I = \log_2 \frac{p'}{p}$$

where p is the probability of the message to be sent, p' is linked to the use that the observer takes from the message and coincides with the probability that the information content, expected from the observer, has to be realized after the transmission of he message.

3 Entropy

In information theory [1] (and in connection with signal theory [2]), entropy measures the quantity of uncertainty of information present in an aleatory signal.
For every transmitted symbol (in a message) there can be a certain quantity of information linked to that symbol.
In the majority of practical applications of information theory, it is needed to operate a choice between messages of a set, each one with its own probability to be sent.
Shannon gave an entropy definition of this set, identifying it with an information content that the choice of one of the messages will send.
If every message has p^1 probability to be sent, entropy is equal to the sum in the entire range of the function $log_2\ p^1/p$, each one specific to a message i.e.

$$H(X) = E[I(x_i)] = \sum_{i=1}^{n} I(x_i) \bullet P(x_i) = \sum_{i=1}^{n} P(x_i) \bullet \log_2 \frac{1'}{P(x_i)}$$

The word “entropy”, borrowed from thermodynamics, indicates therefore the average information content of a message.
According to what has been said until now, the musical message can be defined as a sequence of signals organized according to a code.

4 Information of the musical message

To compare various segments among them, in order to determine which is more important, each entropy is calculated: the less the entropy value, the greater the information carried by the segment.
To calculate entropy, it is necessary to refer to a specific alphabet: it is specific to the language, and, how it can be deduced immediately from the formula (based on the probability that some signals are sent rather than others), it is linked to the language.
In the case of musical language, it has been designed a classification of different melodic intervals like alphabet symbols.
Interval classification consists of *denomination* (generic indication) and *qualification* (specific indication):

denomination corresponds to the number of grades that interval contains, calculated from bass to treble, it can be of 2nd, 3th, 4th, 5th and so on

qualification is deduced from the number of tones and semitones that the interval contains; it can be right, major, minor, ...

Musical language, for research, has to be considered relative to a historical period, to the composer and to the music form of the piece chosen like the subject of automatic analysis, as well as many other factors that can modify the rules (artistic maturity of the author, commission type, social context and so on).
Musical language determinates the presence of specific symbols (intervals) that come from the rules that the composers of that time used to learn at school, from the genius of every composer, from the different type of requested phrasing and from the typical “writing” of different instruments.
The succession of different intervals gives birth the melody, that in the first moment owes the difference between consonance and dissonance to the rules.

[1] With the term “uncertainty”, we intend simply the difficulty of guessing the term.

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The difference of style and taste between two compositions is also due to the different grammatical images that the musician uses in their compositions.

This classification can create too many parameters if we consider it from this point of view, furthermore it could create at the same time a problem coding the knowledgebase for expert systems. With reference a very specific system, so it is chosen to consider "language" as a single composition, using as a general rule the classification of melodic intervals, present in the composition teaching.

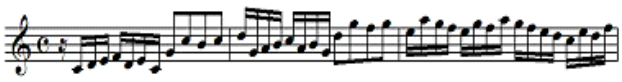
From each single piece it can be extracted the characteristic of the language that has been developed, analysing later the distribution of the intervals.
For every single piece it is filled a table representing its own alphabet, considering for every interval including its own, its ascending or descending trend: figure 6 shows an example of the invention with two voices of BACH n1 in D Major BWV 772 relative to the first staff only.

Interval	Direction	Semi-Tones	N°
Unisono		0	8
2° m	ascending	1	24
	descending	-1	25
2° M	ascending	2	45
	descending	-2	56
3° m	ascending	3	18
	descending	-3	19
3° M	ascending	4	9
	descending	-4	9
4 G	ascending	5	5
	descending	-5	2
4° A / 5° d	ascending	6	1
	descending	-6	1
5° G	ascending	7	2
	descending	-7	3
6° m	ascending	8	0
	descending	-8	4
6° M	ascending	9	2
	descending	-9	0
7° m	ascending	10	2
	descending	-10	1
7° M	ascending	11	0
	descending	-11	0
8° G	ascending	12	2
	descending	-12	0

Fig.6: Alphabet example. The first column indicates the denomination of the interval (classification), the second the ascending or descending movement and the third the number of semi-tones composing an interval (qualification) and the fourth represents the numbers of semi-tones found in that piece.

The determination of such a characterized alphabet is not enough to calculate entropy in a music piece: it is necessary to consider how intervals follow one another inside the piece.
This is the reason why Markov's process is used (or stochastic markovian process): it is a choice to extract transition probability that determines the passage from a system state to the following one only from the immediately previous state.
This determines the loss of a "historical" view of the process focused on a higher number of transitions, but results a good compromise to the need of simplifying the analysis, as well as to respond to composition rules defining a composition of an interval sequence representing the melodic movement towards the note which is added each time.
Surely the characteristic of human creativity considers influential a certain number of intervals previous to that generated, but it is considered as sufficient as the generated model.
Moreover, being the language from which to pull out rules restricted to a specific composition, a multiple-transitions stochastic process could extrapolate its own probabilities from a number of cases too limited to be significant.

So we create the *transition matrix,* built on the transition possibilities between system states *(conditioned probabilities).* In our case, matrix represents chances that a certain interval could resolve in another interval type.
To better explain, let's consider the following example (Fig.1)



In the score we consider the first interval do-re; this corresponds to a major interval (2M), ascending (a), that precedes re-mi interval, that is again a 2nd major interval.
Transition matrix will be updated, incrementing by one unit, the corresponding box at the crossing of the two intervals.

Interval			2m		2M		3m		3M		4G		4A / 5d	
	semi-tones		1		2		3		4		5		6	
			a	d	a	d	a	d	a	d	a	d	a	d
2m	1	a d												
2M	2	a d			1									
3m	3	a d												
3M	4	a d												
4G	5	a d												

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Now, considering this last interval, we can notice that following mi-fa is a 2nd minor interval (2m), ascending (a), so we update matrix incrementing one unit the corresponding box at the crossing between 2nd major ascending intervals with 2nd minor ascending intervals.

Interval	semi-tones		2m	2M	3m	3M	4G	4A / 5
			1	2	3	4	5	6
			a d	a d	a d	a d	a d	a d
2m	1	a						
		d						
2M	2	a	1	1				
		d						
3m	3	a						
		d						
3M	4	a						
		d						
4G	5	a						
		d						

This procedure repeats itself up to the end of the score (see Fig.7).
Note how the configuration in statistical values present in the flowchart is a visual pointer of how alphabet chances can divide the musical piece, and act as a pointer to the redundance in the melodic articulations.

Interval	semi-tones		2m	2M	3m	3M	4G	4A / 5
			1	2	3	4	5	6
			a d	a d	a d	a d	a d	a d
2m	1	a		11	9			
		d	2	1 10	11			
2M	2	a	14	11 1	6	9	1	
		d	14	1 14	7 3	9	1	
3m	3	a	7	11				
		d	6	9 2			1	1
3M	4	a		9				
		d	1	4			1	
4G	5	a	1	2			1	
		d	1		1			
4A / 5d	6	a	1					
		d						
5G	7	a					2	
		d		1				1
6m	8	a						
		d		2 2				
6M	9	a	2					
		d						
7m	10	a		1				
		d		1				
7M	11	a						
		d						
8G	12	a						
		d		2				
concordant				1				

Fig.7: Example of a Transition matrix taken from Bach's “Two voices inventions in D major BWV 772”, relative to only first staff.

5 Obtained results

The analysis model shown in this article has been tested through the creation of an algorithm[2], whose structure considers first of all each of the singular aspects as shown above; algorithm does not consider any limit regarding to table dimensions representative of the alphabet and of the Transition matrix, on the contrary, they will be dimensioned automatically at each analysis, based on the characteristics of the musical piece.
The type of intervals considered is not dimensioned a priori drawing on the knowledge of the analysis of music literature to give algorithm particularity and specificity at every single analysis, thanks to statistics on intervals type present.
Every formal expectation on type of intervals, expected as significant, could restrict analysis to pieces in a certain period, e.g. Baroque (in which the extension of a melodic interval normally does not exceed two octaves), but it would behave in a limited way in XIX-XX century pieces, in which in some instrumental writings the melodic interval can also exceed two octaves.
In conclusion, another important aspect, considered to define logical algorithm bases, is the nature of the data: analysis results are shown through numeric values representative of entropy value in every single segment.
On the basis of entropy definition given in previous paragraphs, it is noticeable that never will values be greater than zero, but instead very low values; so it is important to evaluate decimal number, that can determinate, however slightly, the individuation of the more important segments; the greater the number, the greater the analysis precision.
Many pieces of various authors and periods have been analyzed, seeking to give space between different compositive techniques.

Here are analysis results of three pieces :
Bach's “Two voices inventions in do major BWV 772”
L.Van Beethoven's romance for violin and orchestra op.n.50
J. Brahms' intermezzo op.n.119

[2] *Melodic Segmenter*

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Bach's "Two voices invention in D major BWV 772"

The "invention" (taken in this article from the beginning as a model for musical example), is part of a collection of 15 pieces that are short compositions written by Bach for his pupils for a didactic purpose. But although they were created for teaching purposes these are by no means simple style exercises. They are instead small masterpieces of expression.
As one can read in the heading inserted by Bach, the 15 two-voices inventions were composed to introduce to the study of polyphonic execution and of music composition.
Theme exposition (see Fig.8), from which all invention material will follow, has the purpose of making clear the raw material on which the composition is based.

Fig.8: The subject is represented by a semi-phrase including measure 1 and the first 16th of measure 2. It is composed of two clauses.

The first clause ends on first octave of 3rd movement (truncated – or male - cadenza).

The second, in contrast, starts from the note on which it first ends (linked by elision) and ends with a truncated cadenza on first 16th of measure 2. In imitative composition, it is useful to individualize also the subject head: it represents a fragment often replied in an imitative way that can be described as a first melodic subject mode, the first metric foot.
In the following, we report the final table (Fig.9) restored by algorithm, in which in the first two columns the segment starting positions are reported, for its individualization inside the piece, and its length (how many notes it is composed of).

In third column the entropy value of that segment is shown.

In conclusion, in the last four columns are shown the values relative to every segment redundance in its four possible states : original one (O), retrograde (R), inverse (I) and inverse retrograde (IR).

Position beginning segment	Length segment	Entropy	Redundance			
			O	R	I	RI
1	11	0,00001884462605800000	2			
1	7	0,00193549156832700000	10	3	10	3
1	4	0,07691925621636600000	19	22	22	19
1	3	0,21111050126030800000	36	38	38	36
1	2	0,45817591062063200000	69	81	81	69
3	5	0,02084456356113900000	11	14	14	6
3	4	0,05746669625398200000	16	27	27	16
3	3	0,13818148506486400000	24	27	27	19
4	2	0,18132339179788700000	28	27	27	28
8	2	0,03375187186546800000	6	3	3	6
8	3	0,03375187186546800000	3		1	
9	4	0,01345298154532300000	3	1	1	1
9	3	0,03425194634590900000	4	4	1	1
10	4	0,01097531509609100000	4			3
22	4	0,01316085700101800000	2			
57	2	0,05892005274381100000	4	1	1	4
58	4	0,02354946930524500000	2			3
59	3	0,03375187186546800000	3			3

Fig.9: Final table restored by algorithm.

For a better data evaluation, and to appreciate them in their real meaning, we have projected the entropy values on a graph, using as a reference not "uncertainty" represented by it, but instead the information (less uncertainty means more information).

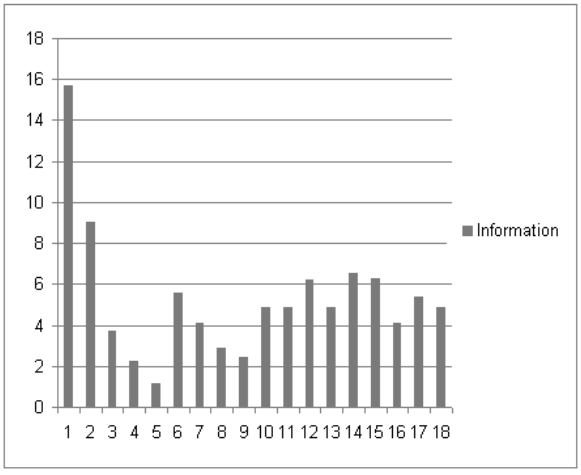

Fig.10: Graphic representation of information carried by various segments found by the segmentation process. The scale used is the logarithmic scale that shows very low values as well as very high ones.
As can be seen, the first segment starting in note 1 and composed by 11 notes corresponds exactly to the main theme.
On the grounds of redundance, the found theme is that one with the lower value (only 2 segments found in original state): let us note that the result is perceptible because major information is not associated to redundance (psycho perceptive approach) but instead to segment structure, that is to its alphabet.

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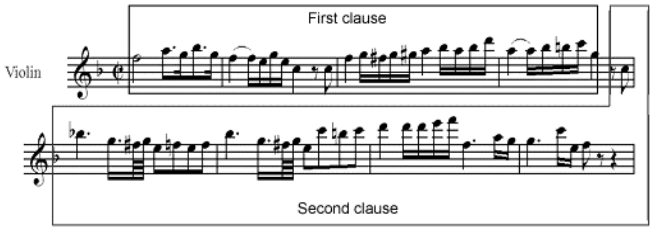
L.Van Beethoven's romance for violin and orchestra op.n.50

For the second piece to analize, it has been chosen a composition written in concerto form, and features a soloist instrument accompanied by an orchestra.
The choice of this piece was suggested for the need to evaluate the strength of algorithm, through a polyphonic piece: the main theme, shown by the solo instrument, (violin), starting from measure 1, in piece representation does not correspond to starting position 1 (like in Bach's piece case).
Moreover, the main theme is composed by 8 measures, that contain two clauses each of 4 measures: first clause starts from measure 1 and ends in measure 4 of 3rd movement; second clause starts from measure 4 of 4th movement and ends in measure 8.



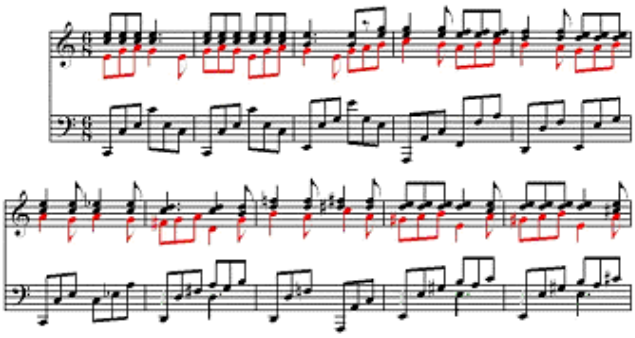
Unlike in the first piece, the list of found segments is very long (383), but the main theme and the second clause emerge as important segments.
Third segment corresponds to solo instrument cadenza, in measure 77, and fourth instead in second phrase, that starts in measure 10 and ends in measure 16.
Here follows a table extract, summarizing the fragments (Fig.11).

Position beginning segment	Length segment	Entropy	Redundance			
			O	R	I	RI
735	54	0,00000000000005000000	3			
763	28	0,00000000000005000000	3			
1425	16	0,00000000000005000000	3			3
519	20	0,00000000000005000000	3			
1276	16	0,00000000000005000000	3			
1607	23	0,00000000000005000000	3			
635	29	0,00000000000009800000	2			
637	29	0,00000000000009800000	2			
1426	17	0,00000000000009800000	2			
1671	16	0,00000000000009800000	3			
768	17	0,00000000000019100000	3			
1035	24	0,00000000000019100000	3			2
1665	19	0,00000000000028300000	3			
881	17	0,00000000000028300000	3			
907	23	0,00000000000028300000	2			
644	26	0,00000000000037500000	3			
163	15	0,00000000000147800000	2			
13	16	0,00000000000186600000	2			

Fig.11: Final table restored by algorithm.

J. Brahms' intermezzo op.119 n. 3

This piece is different from the others given in the accordal type of writing: the perceived melody is almost hidden in the middle of the polyphony.



Also in this case, results satisfy expectations, i.e. the first segment found in the summarizing list (Fig.12) is exactly the one of the identifyi ‿ melody: it has to be mentioned that also in this case the fragment has a very low redundance in relation to the others; that enhances the thesis by which segment structure carries information.
So, the structure of this piece, that characterizes itself in secondary voices with replied accords, could create some problems in seeking segments, if redundance had been considered as the only test parameter.

Position beginning segment	Length segment	Entropy	Redundance			
			O	R	I	RI
540	21	0,00000000000005000000	2			
1	35	0,00000000000009800000	2			
277	36	0,00000000000009800000	2			
115	21	0,00000000014660500000	2			
544	14	0,00000000256360300000	3			
496	10	0,00000000263153000000	2			
540	14	0,00000001893672600000	3			
541	13	0,00000007311031500000	3			
540	13	0,00000008749652400000	3	3		
496	9	0,00000008857401400000	2			2
540	11	0,00000017143621400000	6	3		
542	12	0,00000034373923100000	3	2		
540	12	0,00000036774769100000	3	3		
222	11	0,00000053054711200000	2			
540	10	0,00000101841003700000	7	3		
543	11	0,00000108655389700000	3	3		
212	10	0,00000111082452200000	2			
496	7	0,00000135898484800000	3			2

Fig.12: Final table restored by algorithm.

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6 Conclusions

The instruments shown in this article follow an innovative point of view that lets the discovery and valorisation of information coherent to our perception.

These instruments have been developed focusing on specific musical objectives.

For this reason we have tried to find first of all a unique analysis system, good for every musical form (from fuga to sonata, from romanza to intermezzo) and for every style (from monody to polyphony).

The extension of this methodology to rhythmic analysis of a musical piece could help in the evaluation of the single fragments found.

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